Functions

ESSENTIAL QUESTION

How can you use functions to solve real-world problems?

Real-World Video

Computerized machines can assist doctors in surgeries such as laser vision correction. Each action the surgeon takes results in one end action by the machine. In math, functions also have a one-in-one-out relationship.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Evaluate Expressions

EXAMPLE   Evaluate $3x - 5$ for $x = -2$.

$3x - 5 = 3(-2) - 5$ Substitute the given value of x for x.

$= -6 - 5$ Multiply.

$= -11$ Subtract.

Evaluate each expression for the given value of x.

1. $2x + 3$ for $x = 3$ _____

2. $-4x + 7$ for $x = -1$ _____

3. $1.5x - 2.5$ for $x = 3$ _____

4. $0.4x + 6.1$ for $x = -5$ _____

5. $\frac{2}{3}x - 12$ for $x = 18$ _____

6. $-\frac{5}{8}x + 10$ for $x = -8$ _____

Connect Words and Equations

EXAMPLE   Erik's earnings equal 9 dollars per hour.

e = earnings; h = hours

multiplication

$e = 9 \times h$

Define the variables used in the situation.

Identify the operation involved. "Per" indicates multiplication.

Write the equation.

Define the variables for each situation. Then write an equation.

7. Jana's age plus 5 equals her sister's age.

_____

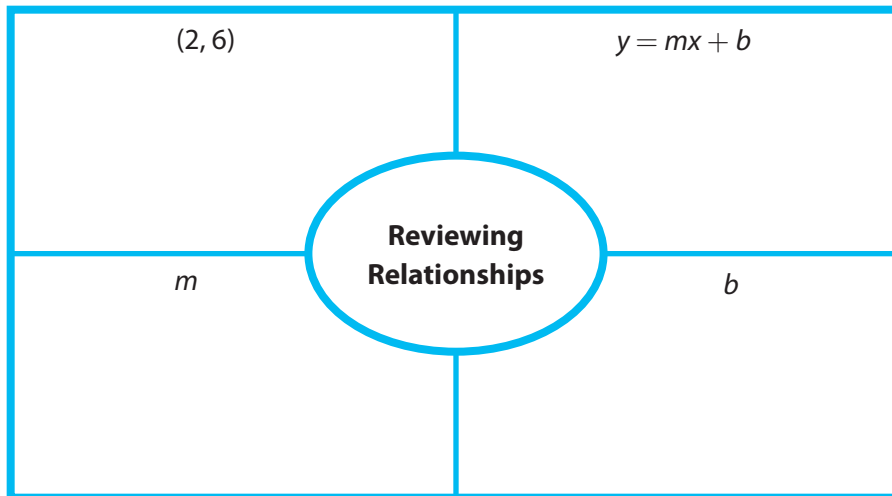**8.** Andrew's class has 3 more students than Lauren's class.

_____

9. The bank is 50 feet shorter than the firehouse.

_____

10. The pencils were divided into 6 groups of 2.

_____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the diagram. You can put more than one word in each section of the diagram.

(2, 6)	$y = mx + b$
m	*b*

Reviewing Relationships

Vocabulary

Review Words
✔ bivariate data *(datos bivariados)*
✔ linear equation *(ecuación lineal)*
  nonlinear relationship *(relación no lineal)*
✔ ordered pair *(par ordenado)*
  proporational relationship *(relación proporcional)*
✔ slope *(pendiente)*
✔ *x*-coordinate *(coordenada x)*
✔ *y*-coordinate *(coordenada y)*
✔ *y*-intercept *(intersección con el eje y)*

Preview Words
  function *(función)*
  input *(valor de entrada)*
  linear function *(función lineal)*
  output *(valor de salida)*

Understand Vocabulary

Complete the sentences using the preview words.

1. A rule that assigns exactly one output to each input

   is a _____.

2. The value that is put into a function is the _____.

3. The result after applying the function machine's rule is

   the _____.

Active Reading

Double-Door Fold Create a double-door fold to help you understand the concepts in this module. Label one flap "Proportional Functions" and the other flap "Non-proportional Functions." As you study each lesson, write important ideas under the appropriate flap. Include any sample problems that will help you remember the concepts when you look back at your notes.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.F.1.1

Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Key Vocabulary

function *(función)*
An input-output relationship that has exactly one output for each input.

What It Means to You

You will identify sets of ordered pairs that are functions. A function is a rule that assigns exactly one output to each input.

UNPACKING EXAMPLE 8.F.1.1

Does the following table of inputs and outputs represent a function?

Yes, it is a function because each number in the input column is assigned to only one number in the output column.

Input	Output
14	110
20	130
22	120
30	110

The graph of the function is the set of ordered pairs (14, 110), (20, 130), (22, 120), and (30, 110).

FL 8.F.1.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

What It Means to You

You will learn to identify and compare functions expressed as equations and tables.

UNPACKING EXAMPLE 8.F.1.2

A spider descends a 20-foot drainpipe at a rate of 2.5 feet per minute. Another spider descends a drainpipe as shown in the table. Find and compare the rates of change and initial values of the linear functions in terms of the situations they model.

Spider #1: $f(x) = -2.5x + 20$

Spider #2:

Time (min)	0	1	2
Height (ft)	32	29	26

For Spider #1, the rate of change is -2.5, and the initial value is 20. For Spider #2, the rate of change is -3, and the initial value is 32.

Spider #2 started at 32 feet, which is 12 feet higher than Spider #1. Spider #1 is descending at 2.5 feet per minute, which is 0.5 foot per minute slower than Spider #2.

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Identifying and Representing Functions

? ESSENTIAL QUESTION

How can you identify and represent functions?

EXPLORE ACTIVITY | Real World

FL 8.F.1.1

Understanding Relationships

Carlos needs to buy some new pencils from the school supply store at his school. Carlos asks his classmates if they know how much pencils cost. Angela says she bought 2 pencils for $0.50. Paige bought 3 pencils for $0.75, and Spencer bought 4 pencils for $1.00.

Carlos thinks about the rule for the price of a pencil as a machine. When he puts the number of pencils he wants to buy into the machine, the machine applies a rule and tells him the total cost of that number of pencils.

Input ➡️   ➡️ Output

	Number of Pencils	Rule	Total Cost
i.	2	?	
ii.	3	?	
iii.	4	?	
iv.	x		
v.	12		

A Use the prices in the problem to fill in total cost in rows **i–iii** of the table.

B Describe any patterns you see. Use your pattern to determine the cost of 1 pencil.

_____

_____

_____

C Use the pattern you identified to write the rule applied by the machine. Write the rule as an algebraic expression and fill in rule column row **iv** of the table.

D Carlos wants to buy 12 pencils. Use your rule to fill in row **v** of the table to show how much Carlos will pay for 12 pencils.

Reflect

1. How did you decide what operation to use in your rule?

_____

_____

2. **What If?** Carlos decides to buy erasers in a package. There are 6 pencil-top erasers in 2 packages of erasers.

   a. Write a rule in words for the number of packages Carlos needs to buy to get *x* erasers. Then write the rule as an algebraic expression.

   _____

   _____

   b. How many packages does Carlos need to buy to get 18 erasers?

   _____

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Identifying Functions from Mapping Diagrams

A **function** assigns exactly one output to each input. The value that is put into a function is the **input**. The result is the **output**.
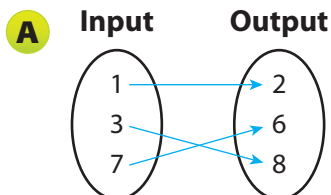
A mapping diagram can be used to represent a relationship between input values and output values. A mapping diagram represents a function if each input value is paired with only one output value.
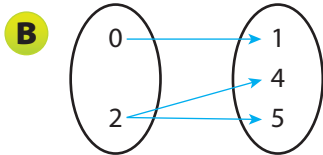
EXAMPLE 1

FL   8.F.1.1

Determine whether each relationship is a function.

A
Input   Output

1   2
3   6
7   8

Since each input value is paired with only one output value, the relationship is a function.

Determine whether each relationship is a function.

B

0 → 1
   → 4
2 → 5

Since 2 is paired with more than one output value (both 4 and 5), the relationship is not a function.
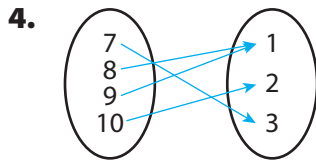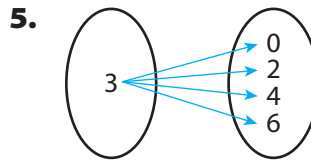
Reflect

3. Is it possible for a function to have more than one input value but only one output value? Provide an illustration to support your answer.
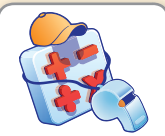
_____

Determine whether each relationship is a function. Explain.

4.

7
8
9
10

1
2
3

_____
_____
_____

5.

3

0
2
4
6

_____
_____
_____

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Math Talk
Mathematical Practices

What is always true about a mapping diagram that represents a function?

Identifying Functions from Tables

Relationships between input values and output values can also be represented using tables. The values in the first column are the input values. The values in the second column are the output values. The relationship represents a function if each input value is paired with only one output value.

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EXAMPLE 2

FL 8.F.1.1

My Notes

Determine whether each relationship is a function.

A

Input	Output
5	7
10	6
15	15
20	2
25	15

Since 15 is a repeated output value, one output value is paired with two input values. If this occurs in a relationship, the relationship can still be a function.

Since each input value is paired with only one output value, the relationship is a function.

Determine whether each relationship is a function.

B

Input	Output
1	10
5	8
4	6
1	4
7	2

> Since 1 is a repeated input value, one input value is paired with two output values. Look back at the rule for functions. Is this relationship a function?

Since the input value 1 is paired with more than one output value (both 10 and 4), the relationship is not a function.

Reflect

6. What is always true about the numbers in the first column of a table that represents a function? Why must this be true?
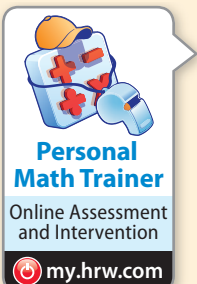
_____

_____

_____

_____

_____

YOUR TURN

Determine whether each relationship is a function. Explain

7.

Input	Output
53	53
24	24
32	32
17	17
45	45

_____

_____

_____

_____

8.

Input	Output
14	52
8	21
27	16
36	25
8	34

_____

_____

_____

_____

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Identifying Functions from Graphs

Graphs can be used to display relationships between two sets of numbers. Each point on a graph represents an ordered pair. The first coordinate in each ordered pair is the input value. The second coordinate is the output value. The graph represents a function if each input value is paired with only one output value.

EXAMPLE 3

FL 8.F.1.1

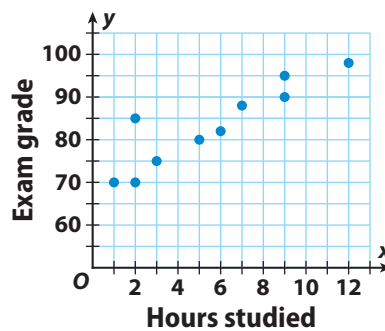**The graph shows the relationship between the number of hours students spent studying for an exam and the exam grades. Is the relationship represented by the graph a function?**

The input values are the number of hours spent studying by each student. The output values are the exam grades. The points represent the following ordered pairs:

Hours Studied and Exam Grade

(1, 70)	(2, 70)	(2, 85)	(3, 75)	(5, 80)
(6, 82)	(7, 88)	(9, 90)	(9, 95)	(12, 98)

Notice that 2 is paired with both 70 and 85, and 9 is paired with both 90 and 95. Therefore, since these input values are paired with more than one output value, the relationship is not a function.

Reflect

9. Many real-world relationships are functions. For example, the amount of money made at a car wash is a function of the number of cars washed. Give another example of a real-world function.

_____

_____

YOUR TURN

10. The graph shows the relationship between the heights and weights of the members of a basketball team. Is the relationship represented by the graph a function? Explain.

Heights and Weights of Team Members
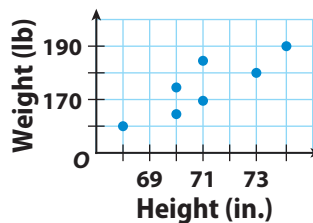
_____

_____

_____

Complete each table. In the row with _x_ as the input, write a rule as an algebraic expression for the output. Then complete the last row of the table using the rule. (Explore Activity)

1.

Input	Output
Tickets	Cost ($)
2	40
5	100
7	140
_x_	
10	

2.

Input	Output
Minutes	Pages
2	1
10	5
20	10
_x_	
30	

3.

Input	Output
Muffins	Cost ($)
1	2.25
3	6.75
6	13.50
_x_	
12	

Determine whether each relationship is a function. (Examples 1 and 2)

4.

_____

_____

_____
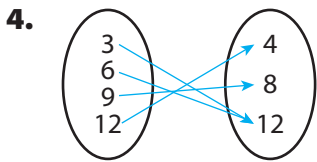
5.

Input	Output
3	20
4	25
5	30
4	35
6	40

_____

_____

_____

6. The graph shows the relationship between the weights of 5 packages and the shipping charge for each package. Is the relationship represented by the graph a function? Explain.

_____

_____

Weights and Shipping Costs
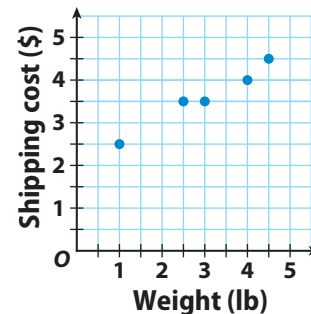
? ESSENTIAL QUESTION CHECK-IN

7. What are four different ways of representing functions? How can you tell if a relationship is a function?

_____

_____

6.1 Independent Practice

FL 8.F.1.1

Determine whether each relationship represented by the ordered pairs is a function. Explain.

8. (2, 2), (3, 1), (5, 7), (8, 0), (9, 1)

9. (0, 4), (5, 1), (2, 8), (6, 3), (5, 9)

10. Draw Conclusions Joaquin receives $0.40 per pound for 1 to 99 pounds of aluminum cans he recycles. He receives $0.50 per pound if he recycles more than 100 pounds. Is the amount of money Joaquin receives a function of the weight of the cans he recycles? Explain your reasoning.

**11.** A biologist tracked the growth of a strain of bacteria, as shown in the graph.

   a. Explain why the relationship represented by the graph is a function.

   b. What If? Suppose there was the same number of bacteria for two consecutive hours. Would the graph still represent a function? Explain.
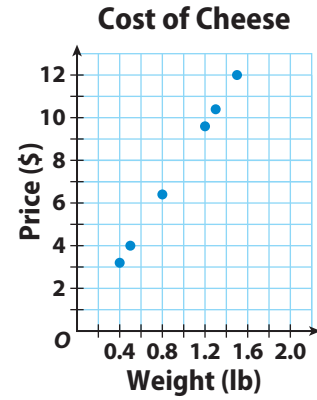
Bacteria B
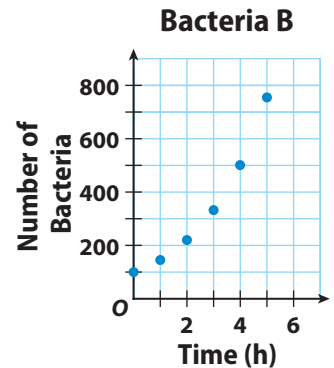
12. Multiple Representations Give an example of a function in everyday life, and represent it as a graph, a table, and a set of ordered pairs. Describe how you know it is a function.

x				
y				
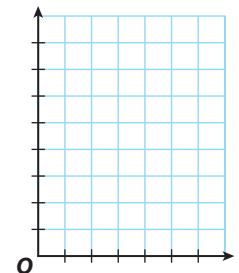
The graph shows the relationship between the weights of six wedges of cheese and the price of each wedge.

13. Is the relationship represented by the graph a function? Justify your reasoning. Use the words "input" and "output" in your explanation, and connect them to the context represented by the graph.

_____

_____

Cost of Cheese

14. **Analyze Relationships** Suppose the weights and prices of additional wedges of cheese were plotted on the graph. Might that change your answer to question 13? Explain your reasoning.

_____

_____

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

15. **Justify Reasoning** A mapping diagram represents a relationship that contains three different input values and four different output values. Is the relationship a function? Explain your reasoning.

_____

_____

_____

_____

16. **Communicate Mathematical Ideas** An onion farmer is hiring workers to help harvest the onions. He knows that the number of days it will take to harvest the onions is a function of the number of workers he hires. Explain the use of the word "function" in this context.

_____

_____

_____

_____

_____

_____

Work Area

6.2 Describing Functions

? ESSENTIAL QUESTION

What are some characteristics that you can use to describe functions?

EXPLORE ACTIVITY Real World   FL 8.F.1.1

Investigating a Constant Rate of Change

The U.S. Department of Agriculture defines heavy rain as rain that falls at a rate of 1.5 centimeters per hour.

A The table shows the total amount of rain that falls in various amounts of time during a heavy rain. Complete the table.

Time (h)	0	1	2	3	4	5
Total Amount of Rain (cm)	0	1.5				

B Plot the ordered pairs from the table on the coordinate plane at the right.

C How much rain falls in 3.5 hours? _____

D Plot the point corresponding to 3.5 hours of heavy rain.

E What do you notice about all of the points you plotted?

_____

F Is the total amount of rain that falls a function of the number of hours that rain has been falling? Why or why not?

_____

_____

Heavy Rainfall

Total Amount of Rain (cm)

Time (h)
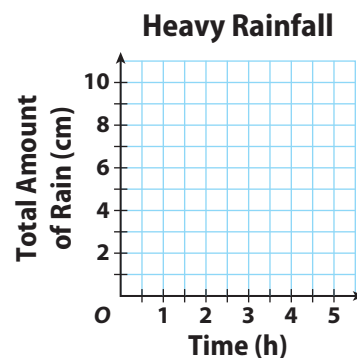
Reflect

1. Suppose you continued to plot points for times between those in the table, such as 1.2 hours or 4.5 hours. What can you say about the locations of these points?

_____

_____

Graphing Linear Functions

The relationship you investigated in the previous activity can be represented by the equation $y = 1.5x$, where x is the time and y is the total amount of rain. The graph of the relationship is a line, so the equation is a **linear equation**. Since there is exactly one value of y for each value of x, the relationship is a function. It is a **linear function** because its graph is a nonvertical line.

EXAMPLE 1 Real World

FL 8.F.1.3

The temperature at dawn was 8 °F and increased steadily 2 °F every hour. The equation $y = 2x + 8$ gives the temperature y after x hours. State whether the relationship between the time and the temperature is proportional or nonproportional. Then graph the function.

STEP 1 Compare the equation with the general linear equation $y = mx + b$.
$y = 2x + 8$ is in the form $y = mx + b$, with $m = 2$ and $b = 8$. Therefore, the equation is a linear equation. Since $b \neq 0$, the relationship is nonproportional.
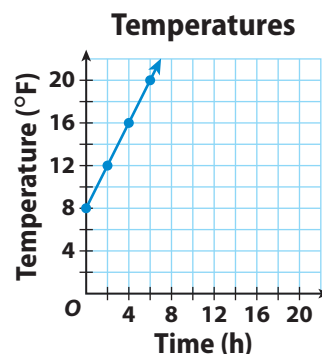
STEP 2 Choose several values for the input x. Substitute these values for x in the equation to find the output y.

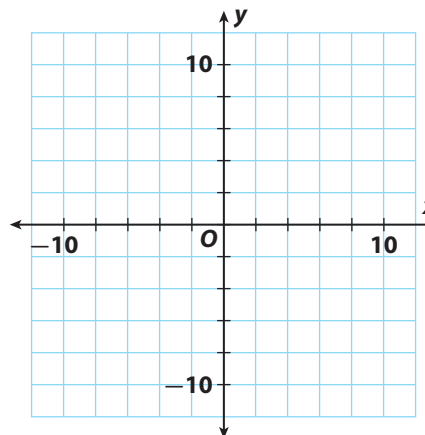| x	$2x + 8$	y	(x, y)
0	$2(0) + 8$	8	$(0, 8)$
2	$2(2) + 8$	12	$(2, 12)$
4	$2(4) + 8$	16	$(4, 16)$
6	$2(6) + 8$	20	$(6, 20)$

STEP 3 Graph the ordered pairs. Then draw a line through the points to represent the solutions of the function.

Temperatures

YOUR TURN

2. State whether the relationship between x and y in $y = 0.5x$ is proportional or nonproportional. Then graph the function.

_____

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Determining Whether a Function is Linear

The linear equation in Example 1 has the form $y = mx + b$, where m and b are real numbers. Every equation in the form $y = mx + b$ is a linear equation. The linear equations represent linear functions. Equations that cannot be written in this form are not linear equations, and therefore are not linear functions.

EXAMPLE 2 · Real World

FL · 8.F.1.3

A square tile has a side length of x inches. The equation $y = x^2$ gives the area of the tile in square inches. Determine whether the relationship between x and y is linear and, if so, if it is proportional.

STEP 1 Choose several values for the input x. Substitute these values for x in the equation to find the output y.

| x	x^2	y	(x, y)
1	1^2	1	$(1, 1)$
2	2^2	4	$(2, 4)$
3	3^2	9	$(3, 9)$
4	4^2	16	$(4, 16)$

STEP 2 Graph the ordered pairs.

STEP 3 Identify the shape of the graph. The points suggest a curve, not a line. Draw a curve through the points to represent the solutions of the function.

STEP 4 Describe the relationship between x and y.

The graph is not a line so the relationship is not linear.

Only a linear relationship can be proportional, so the relationship is not proportional.
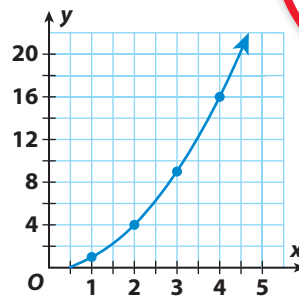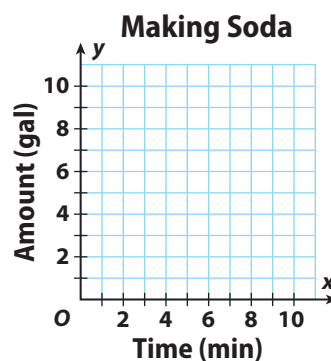

Math Talk
Mathematical Practices

How can you use the numbers in the table to decide whether or not the relationship between x and y is linear?

YOUR TURN

3. A soda machine makes $\frac{2}{3}$ gallon of soda every minute. The total amount y that the machine makes in x minutes is given by the equation $y = \frac{2}{3}x$. Determine whether the relationship between x and y is linear and, if so, if it is proportional.

Time (min), x	0	3		9
Amount (gal), y			4	

Making Soda

Amount (gal) / Time (min)

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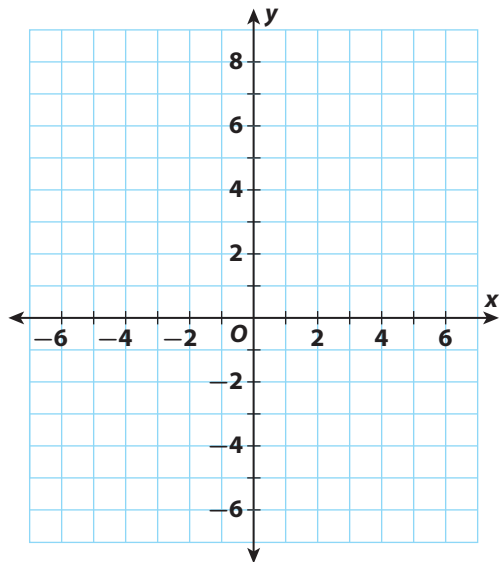
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Lesson 6.2 **163**

Plot the ordered pairs from the table. Then graph the function represented by the ordered pairs and tell whether the function is linear or nonlinear. (Examples 1 and 2)
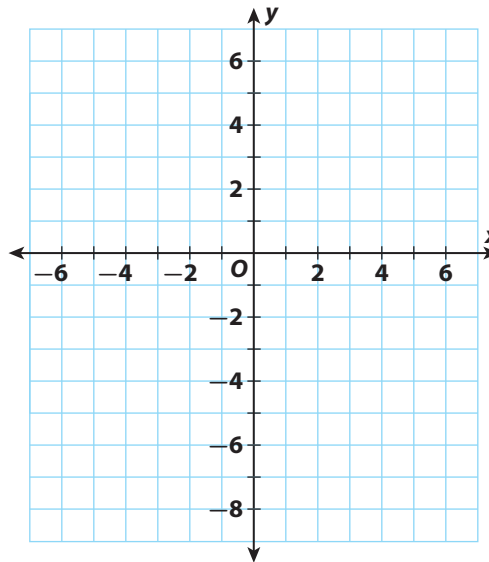
1. $y = 5 - 2x$

Input, x	−1	1	3	5
Output, y				

_____

2. $y = 2 - x^2$

Input, x	−2	−1	0	1	2
Output, y					

_____

Explain whether each equation is a linear equation. (Example 2)

3. $y = x^2 - 1$

_____

_____

_____

_____

4. $y = 1 - x$

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

5. Explain how you can use a table of values, an equation, and a graph to determine whether a function represents a proportional relationship.

_____

_____

_____

6.2 Independent Practice

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6. State whether the relationship between x and y in $y = 4x - 5$ is proportional or nonproportional. Then graph the function.

_____

7. The Fortaleza telescope in Brazil is a radio telescope. Its shape can be approximated with the equation $y = 0.013x^2$. Is the relationship between x and y linear? Is it proportional? Explain.

_____

_____

8. Kiley spent $20 on rides and snacks at the state fair. If x is the amount she spent on rides, and y is the amount she spent on snacks, the total amount she spent can be represented by the equation $x + y = 20$. Is the relationship between x and y linear? Is it proportional? Explain.

_____

_____

_____
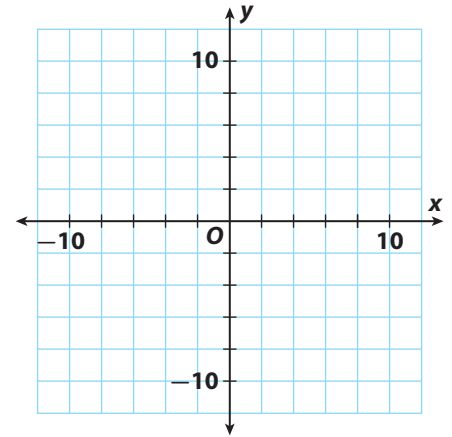
9. Represent Real-World Problems The drill team is buying new uniforms. The table shows y, the total cost in dollars, and x, the number of uniforms purchased.

Drill Team Uniforms

Number of uniforms, x	1	3	5	9
Total cost ($), y	60	180	300	540

a. Use the data to draw a graph. Is the relationship between x and y linear? Explain.
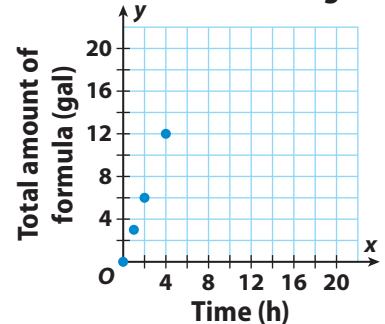
_____

b. Use your graph to predict the cost of purchasing 12 uniforms.

_____

10. Marta, a whale calf in an aquarium, is fed a special milk formula. Her handler uses a graph to track the number of gallons of formula y the calf drinks in x hours. Is the relationship between x and y linear? Is it proportional? Explain.

_____

_____

Marta's Feedings

© Houghton Mifflin Harcourt Publishing Company
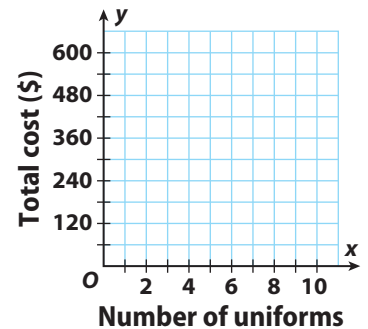
11. Critique Reasoning A student claims that the equation $y = 7$ is not a linear equation because it does not have the form $y = mx + b$. Do you agree or disagree? Why?

_____

_____

_____

12. Make a Prediction Let x represent the number of hours you read a book and y represent the total number of pages you have read. You have already read 70 pages and can read 30 pages per hour. Write an equation relating x hours and y pages you read. Then predict the total number of pages you will have read after another 3 hours.

_____

Work Area

13. Draw Conclusions Rebecca draws a graph of a real-world relationship that turns out to be a set of unconnected points. Can the relationship be linear? Can it be proportional? Explain your reasoning.

_____

_____

_____

14. Communicate Mathematical Ideas Write a real-world problem involving a proportional relationship. Explain how you know the relationship is proportional.

_____

_____

_____

_____

15. Justify Reasoning Show that the equation $y + 3 = 3(2x + 1)$ is linear and that it represents a proportional relationship between x and y.

_____

_____

_____

_____

_____

LESSON

6.3 Comparing Functions

FL 8.F.1.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *Also 8.EE.2.5, 8.F.2.4*

? **ESSENTIAL QUESTION**

How can you use tables, graphs, and equations to compare functions?

Comparing a Table and an Equation

To compare a function written as an equation and another function represented by a table, find the equation for the function in the table.

Math On the Spot
my.hrw.com

EXAMPLE 1 · Real World

FL 8.F.1.2, 8.F.2.4

Josh and Maggie buy MP3 files from different music services. The monthly cost, *y* dollars, for *x* songs is linear. The cost of Josh's service is $y = 0.50x + 10$. The cost of Maggie's service is shown below.

Monthly Cost of MP3s at Maggie's Music Service					
Songs, *x*	5	10	15	20	25
Cost ($), *y*	4.95	9.90	14.85	19.80	24.75

A Write an equation to represent the monthly cost of Maggie's service.

STEP 1 Choose any two ordered pairs from the table to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9.90 - 4.95}{10 - 5} = \frac{4.95}{5} = 0.99$$

The points (5, 4.95) and (10, 9.90) were used.

STEP 2 Find the *y*-intercept. Use the slope and any point.

$y = mx + b$ *Slope-intercept form.*

$4.95 = 0.99 \cdot 5 + b$ *Substitute for y, m, and x.*

$0 = b$

STEP 3 Substitute the slope and *y*-intercept.
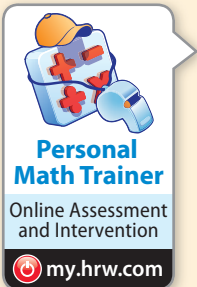
$y = 0.99x + 0$ or $y = 0.99x$ *Substitute 0.99 for m and 0 for b.*

Math Talk
Mathematical Practices

Describe each service's cost in words using the meanings of the slopes and *y*-intercepts.

B Which service is cheaper when 30 songs are downloaded?

Josh's service: Maggie's service:

$y = 0.50 \times 30 + 10$ $y = 0.99 \times 30$

$y = 25$ $y = 29.7$

Josh's service is cheaper.

YOUR TURN

1. Quentin is choosing between buying books at the bookstore or buying online versions of the books for his tablet. The cost, y dollars, of ordering books online for x books is $y = 6.95x + 1.50$. The cost of buying the books at the bookstore is shown in the table. Which method of buying books is more expensive if Quentin wants to buy 6 books?

Cost of Books at the Bookstore					
Books, x	1	2	3	4	5
Cost ($), y	7.50	15.00	22.50	30.00	37.50

_____

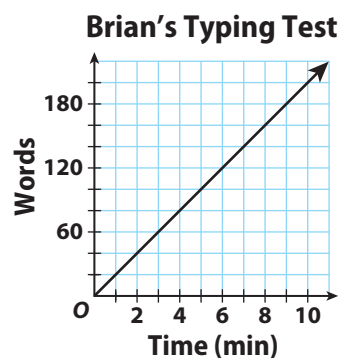## EXPLORE ACTIVITY 1   Real World

FL   8.F.1.2, 8.EE.2.5

Comparing a Table and a Graph

The table and graph show how many words Morgan and Brian typed correctly on a typing test. For both students, the relationship between words typed correctly and time is linear.

Brian's Typing Test

Morgan's Typing Test					
Time (min)	2	4	6	8	10
Words	30	60	90	120	150

A Find Morgan's unit rate.

_____

B Find Brian's unit rate.

_____

C Which student types more correct words per minute?

_____

Reflect

2. Katie types 17 correct words per minute. Explain how a graph of Katie's test results would compare to Morgan's and Brian's.

_____

_____

Comparing a Graph and a Description

Jamal wants to buy a new game system that costs $200. He does not have enough money to buy it today, so he compares layaway plans at different stores.

The plan at Store A is shown on the graph.

Store B requires an initial payment of $60 and weekly payments of $20 until the balance is paid in full.

Layaway Plan

y-axis: Balance owed ($) — 40, 80, 120, 160
x-axis: Number of weeks — 2, 4, 6, 8, 10
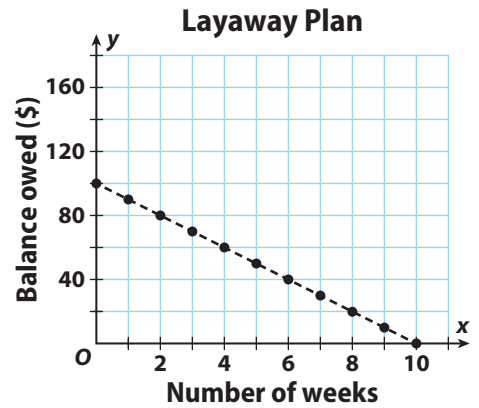
A Write an equation in slope-intercept form for Store A's layaway plan. Let *x* represent number of weeks and *y* represent balance owed.

_____

B Write an equation in slope-intercept form for Store B's layaway plan. Let *x* represent number of weeks and *y* represent balance owed.

_____

C Sketch a graph of the plan at Store B on the same grid as Store A.

D How can you use the graphs to tell which plan requires the greater down payment? How can you use the equations?

_____

_____

_____

E How can you use the graphs to tell which plan requires the greater weekly payment?

_____

_____

_____

F Which plan allows Jamal to pay for the game system faster? Explain.

_____

_____

Doctors have two methods of calculating maximum heart rate. With the first method, maximum heart rate, y, in beats per minute is $y = 220 - x$, where x is the person's age. Maximum heart rate with the second method is shown in the table. (Example 1)

Age, x	20	30	40	50	60
Heart rate (bpm), y	194	187	180	173	166

1. Which method gives the greater maximum heart rate for a 70-year-old?

_____

2. Are heart rate and age proportional or nonproportional for each method?

_____

Aisha runs a tutoring business. With Plan 1, students may choose to pay $15 per hour. With Plan 2, they may follow the plan shown on the graph. (Explore Activity 1 and 2)

Tutoring Fees
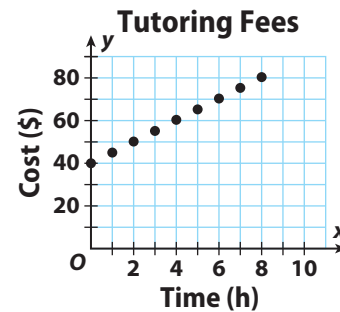
3. Describe the plan shown on the graph.

_____

4. Sketch a graph showing the $15 per hour option.

5. What does the intersection of the two graphs mean?

_____

6. Which plan is cheaper for 10 hours of tutoring?

_____

7. Are cost and time proportional or nonproportional for each plan?

_____

_____

? ESSENTIAL QUESTION CHECK-IN

8. When using tables, graphs, and equations to compare functions, why do you find the equations for tables and graphs?
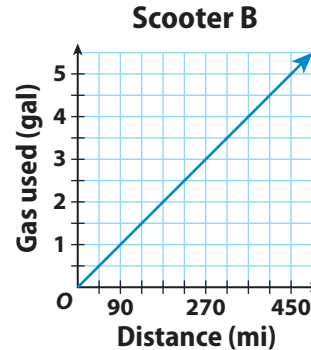
_____

_____

6.3 Independent Practice

FL 8.EE.2.5, 8.F.1.2, 8.F.2.4

Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

The table and graph show the miles driven and gas used for two scooters.

Scooter A	
Distance (mi), x	Gas used (gal), y
150	2
300	4
450	6
600	8
750	10

Scooter B

9. Which scooter uses fewer gallons of gas when 1350 miles are driven?

_____

10. Are gas used and miles proportional or nonproportional for each scooter?

_____

A cell phone company offers two texting plans to its customers. The monthly cost, y dollars, of one plan is $y = 0.10x + 5$, where x is the number of texts. The cost of the other plan is shown in the table.

| Number of texts, x	100	200	300	400	500
Cost ($), y	20	25	30	35	40

11. Which plan is cheaper for under 200 texts? _____

12. The graph of the first plan does not pass through the origin. What does this indicate?

_____

_____

13. Brianna wants to buy a digital camera for a photography class. One store offers the camera for $50 down and a payment plan of $20 per month. The payment plan for a second store is described by $y = 15x + 80$, where y is the total cost in dollars and x is the number of months. Which camera is cheaper when the camera is paid off in 12 months? Explain.

_____

_____

14. The French club and soccer team are washing cars to earn money. The amount earned, y dollars, for washing x cars is a linear function. Which group makes the most money per car? Explain.

French Club	
Number of cars, x	Amount earned ($), y
2	10
4	20
6	30
8	40
10	50

Soccer Team

_____

_____

_____
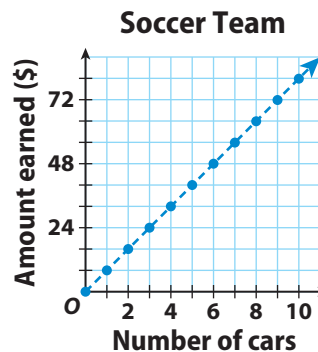
![H.O.T.] **FOCUS ON HIGHER ORDER THINKING**

15. Draw Conclusions Gym A charges $60 a month plus $5 per visit. The monthly cost at Gym B is represented by $y = 5x + 40$, where x is the number of visits per month. What conclusion can you draw about the monthly costs of the gyms?

_____

_____

_____

16. Justify Reasoning Why will the value of y for the function $y = 5x + 1$ always be greater than that for the function $y = 4x + 2$ when $x > 1$?

_____

_____

_____

17. Analyze Relationships The equations of two functions are $y = -21x + 9$ and $y = -24x + 8$. Which function is changing more quickly? Explain.

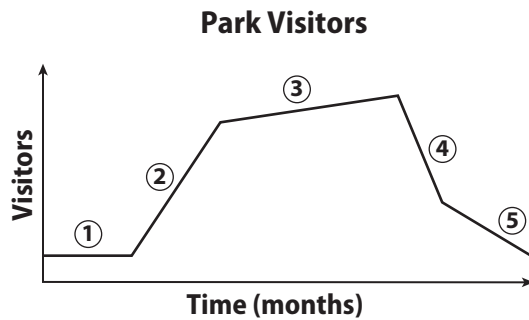_____

_____

_____

_____

LESSON
6.4 Analyzing Graphs

? ESSENTIAL QUESTION

How can you describe a relationship given a graph and sketch a graph given a description?

EXPLORE ACTIVITY 1 — Real World

FL 8.F.2.5

Interpreting Graphs

A roller coaster park is open from May to October each year. The graph shows the number of park visitors over its season.

Park Visitors

A Segment 1 shows that attendance during the opening weeks of the park's season stayed constant. Describe what Segment 2 shows.

_____

B Based on the time frame, give a possible explanation for the change in attendance represented by Segment 2.

_____

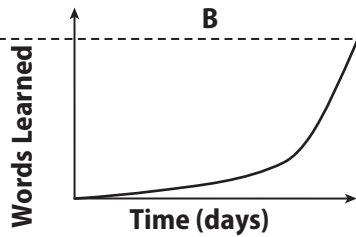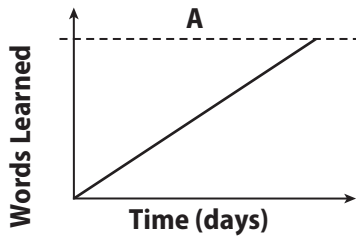**C** Which segments of the graph show decreasing attendance? Give a possible explanation.

_____

_____

Reflect

1. Explain how the slope of each segment of the graph is related to whether attendance increases or decreases.

_____

_____

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Matching Graphs to Situations

Grace, Jet, and Mike are studying 100 words for a spelling bee.

- Grace started by learning how to spell many words each day, but then learned fewer and fewer words each day.
- Jet learned how to spell the same number of words each day.
- Mike started by learning how to spell only a few words each day, but then learned a greater number of words each day.

F-U-N-C-T-I-O-N

Graph A: Words Learned vs Time (days)

Graph B: Words Learned vs Time (days)

Graph C: Words Learned vs Time (days)

_____   _____   _____
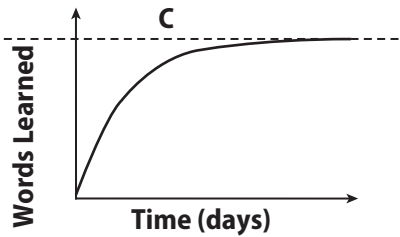
A Describe the progress represented by Graph A.

_____

_____

Math Talk
Mathematical Practices
Tell whether each graph is linear or nonlinear and proportional or nonproportional.

B Describe the progress represented by Graph B.

_____

_____

_____

C Describe the progress represented by Graph C.

_____

_____

_____

D Determine which graph represents each student's study progress and write the students' names under the appropriate graphs.

Reflect

2. What would it mean if one of the graphs slanted downward?

_____

# Sketching a Graph for a Situation

Mrs. Sutton provides free math tutoring to her students every day after school. No one comes to tutoring sessions during the first week of school. Over the next two weeks, use of the tutoring service gradually increases.

A Sketch a graph showing the number of students who use the tutoring service over the first three weeks of school.

Students

O   1   2   3   4   5
Time (weeks)
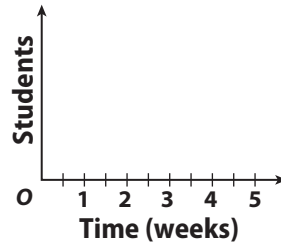
B Mrs. Sutton's students are told that they will have a math test at the end of the fifth week of school. How do you think this will affect the number of students who come to tutoring?

_____

_____

C Considering your answer to **B**, sketch a graph showing the number of students who might use the tutoring service over the first six weeks of school.
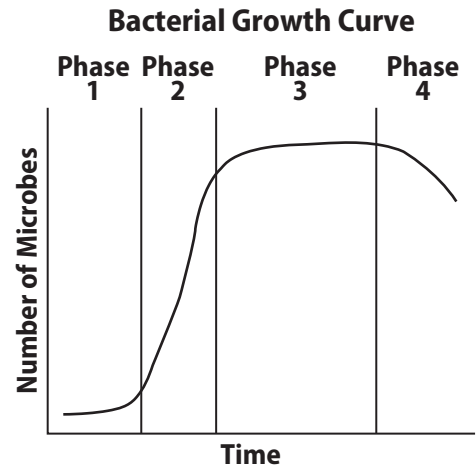
Students

O   1   2   3   4   5   6   7
Time (weeks)
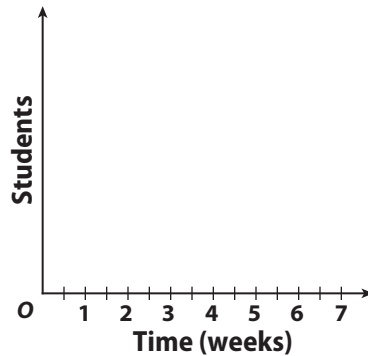
Reflect

3. If Mrs. Sutton offers bonus credit to students who come to tutoring, how might this affect the number of students?

_____

4. How would your answer to Question 3 affect the graph?

_____

_____

In a lab environment, colonies of bacteria follow a predictable pattern of growth. The graph shows this growth over time. (Explore Activity 1)
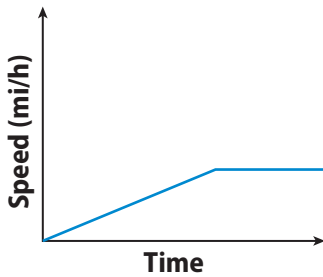
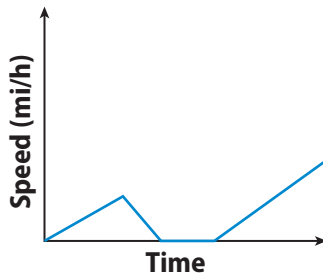**1.** What is happening to the population during Phase 2?

_____

_____

2. What is happening to the population during Phase 4?

_____

_____

Bacterial Growth Curve

The graphs give the speeds of three people who are riding snowmobiles. Tell which graph corresponds to each situation. (Explore Activity 2)
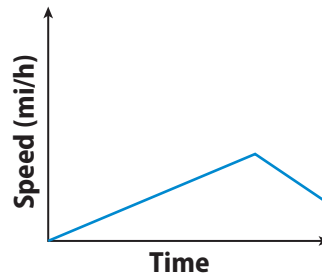
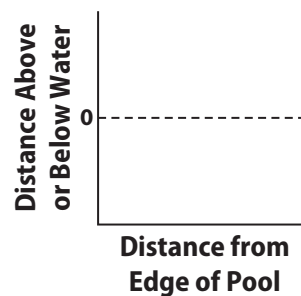**Graph 1**

Graph 2

Graph 3

3. Chip begins his ride slowly but then stops to talk with some friends. After a few minutes, he continues his ride, gradually increasing his speed.

_____

4. Linda steadily increases her speed through most of her ride. Then she slows down as she nears some trees.

_____

5. Paulo stood at the top of a diving board. He walked to the end of the board, and then dove forward into the water. He plunged down below the surface, then swam straight forward while underwater. Finally, he swam forward and upward to the surface of the water. Draw a graph to represent Paulo's elevation at different distances from the edge of the pool. (Explore Activity 3)

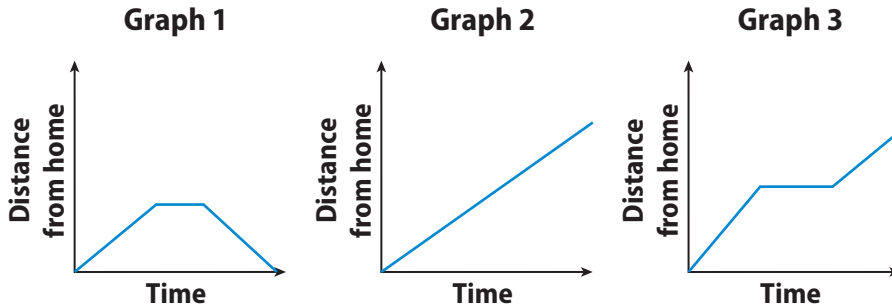**Paulo's elevation**

6.4 Independent Practice

FL 8.F.2.5

Tell which graph corresponds to each situation below.

Graph 1

Distance from home / Time

Graph 2

Distance from home / Time

Graph 3

Distance from home / Time

6. Arnold started from home and walked to a friend's house. He stayed with his friend for a while and then walked to another friend's house farther from home. _____

7. Francisco started from home and walked to the store. After shopping, he walked back home. _____

8. Celia walks to the library at a steady pace without stopping. _____

Regina rented a motor scooter. The graph shows how far away she is from the rental site after each half hour of riding.

9. Represent Real-World Problems Use the graph to describe Regina's trip. You can start the description like this: "Regina left the rental shop and rode for an hour…"

_____

_____

_____

_____

_____

Distance from Rental Site
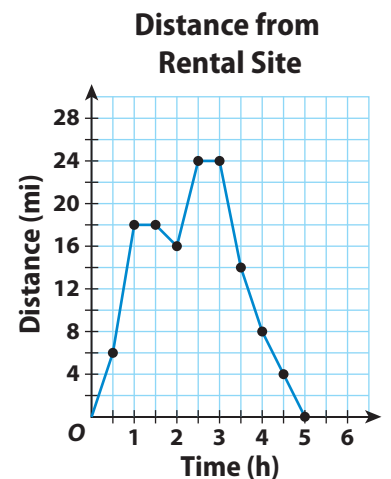
10. Analyze Relationships Determine during which half hour Regina covered the greatest distance.

_____

The data in the table shows the speed of a ride at an amusement park at different times one afternoon.

Time	3:20	3:21	3:22	3:23	3:24	3:25
Speed (mi/h)	0	14	41	62	8	0
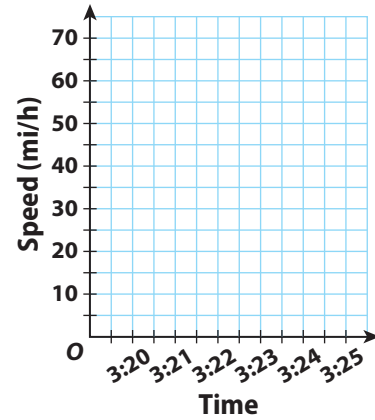
**11.** Sketch a graph that shows the speed of the ride over time.

12. Between which times is the ride's speed increasing the fastest?

_____

13. Between which times is the ride's speed decreasing the fastest?

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

A woodland area on an island contains a population of foxes. The graph describes the changes in the population over time.

14. Justify Reasoning What is happening to the fox population before time *t*? Explain your reasoning.

_____

_____

_____

_____

_____
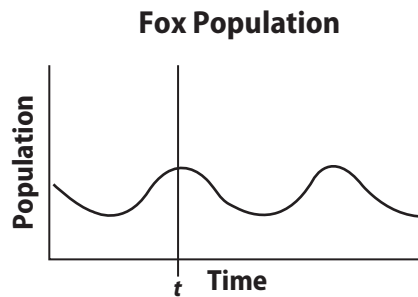
Fox Population

15. What If? Suppose at time *t*, a conservation organization moves a large group of foxes to the island. Sketch a graph to show how this action might affect the population on the island after time *t*.
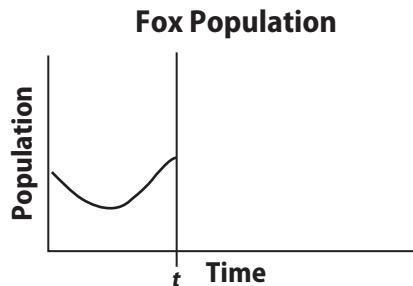
16. Make a Prediction At some point after time *t*, a forest fire destroys part of the woodland area on the island. Describe how your graph from problem 15 might change.
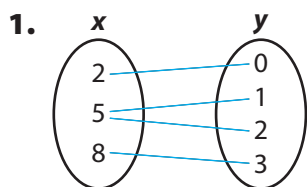
_____

_____

_____

Fox Population

Ready to Go On?
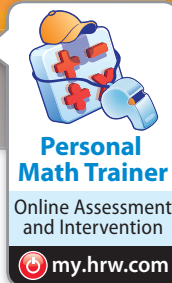
6.1 Identifying and Representing Functions

Determine whether each relationship is a function.

1.

x	y
2	0
5	1
8	2
	3

2.

Input, x	Output, y
−1	6
3	5
6	5

3. (2, 5), (7, 2), (−3, 4), (2, 9), (1, 1)

_____   _____   _____

6.2 Describing Functions

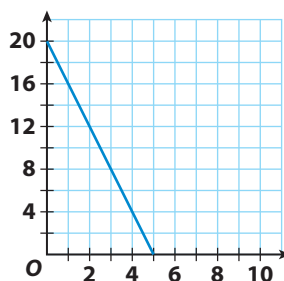**Determine whether each situation is linear or nonlinear, and proportional or nonproportional.**

4. Joanna is paid $14 per hour.

5. Alberto started out bench pressing 50 pounds. He then added 5 pounds every week.

_____   _____

6.3 Comparing Functions

6. Which function is changing more quickly? Explain.

_____

_____

_____

Function 1

Function 2

Input, x	Output, y
2	11
3	6.5
4	2

6.4 Analyzing Graphs

7. Describe a graph that shows Sam running at a constant rate.

_____

? ESSENTIAL QUESTION

8. How can you use functions to solve real-world problems?

_____

Assessment Readiness

Selected Response

1. Which table shows a proportional function?

Ⓐ

x	0	5	10
y	3	15	30

Ⓑ

x	0	5	10
y	10	20	30

Ⓒ

x	0	5	10
y	0	50	100

Ⓓ

x	0	5	10
y	10	5	0

2. What is the slope and y-intercept of the function shown in the table?

x	1	4	7
y	6	12	18

Ⓐ $m = -2; b = -4$

Ⓑ $m = -2; b = 4$

Ⓒ $m = 2; b = 4$

Ⓓ $m = 4; b = 2$

3. The table below shows some input and output values of a function.

Input	4	5	6	7
Output	14	17.5		24.5

What is the missing output value?

Ⓐ 20

Ⓑ 21

Ⓒ 22

Ⓓ 23

4. Tom walked to school at a steady pace, met his sister, and they walked home at a steady pace. Describe this graph.

Ⓐ V-shaped

Ⓑ upside down V-shaped

Ⓒ Straight line sloping up

Ⓓ Straight line sloping down

Mini-Task

5. Linear functions can be used to find the price of a building based on its floor area. Below are two of these functions.
$y = 40x + 15,000$

Floor Area (ft²)	400	700	1,000
Price ($1,000s)	32	56	80

a. Find and compare the slopes.

_____

_____

_____

b. Find and compare the y-intercepts.

_____

_____

_____

c. Describe each function as proportional or nonproportional.

_____

_____

_____

Study Guide Review

MODULE 3 — Proportional Relationships

Key Vocabulary
constant of proportionality *(constante de proporcionalidad)*
proportional relationship *(relación proporcional)*
slope *(pendiente)*

? ESSENTIAL QUESTION
How can you use proportional relationships to solve real-world problems?

EXAMPLE 1
Write an equation that represents the proportional relationship shown in the graph.
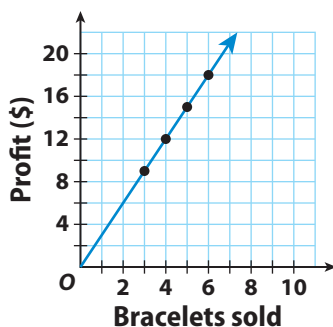
Use the points on the graph to make a table.

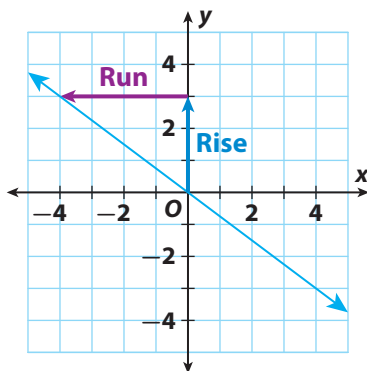| Bracelets sold	3	4	5	6
Profit ($)	9	12	15	18

Let *x* represent the number of bracelets sold.

Let *y* represent the profit.

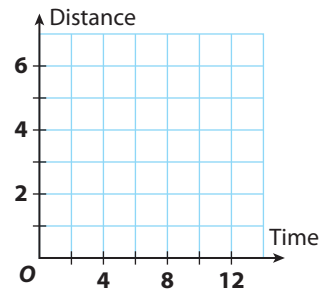The equation is $y = 3x$.

EXAMPLE 2
Find the slope of the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

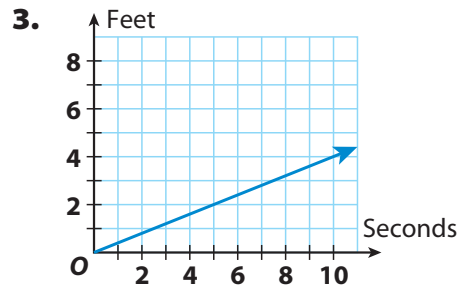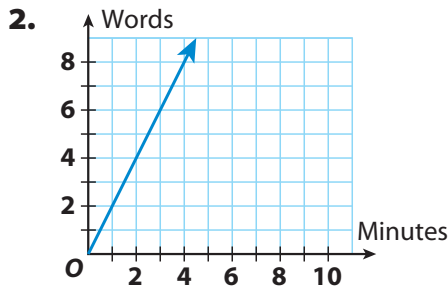$$= \frac{3}{-4}$$

$$= -\frac{3}{4}$$

EXERCISES

1. The table represents a proportional relationship. Write an equation that describes the relationship. Then graph the relationship represented by the data. (Lessons 3.1, 3.3, 3.4)

Time (x)	6	8	10	12
Distance (y)	3	4	5	6

_____

Find the slope and the unit rate represented on each graph. (Lesson 3.2)

2.

_____

3.

_____

MODULE **4**

Nonproportional Relationships

Key Vocabulary

linear equation
  (ecuación lineal)
slope-intercept form of an
  equation *(forma de
  pendiente-intersección)*
y-intercept
  (intersección con el eje y)

? ESSENTIAL QUESTION

How can you use nonproportional relationships to solve real-world problems?
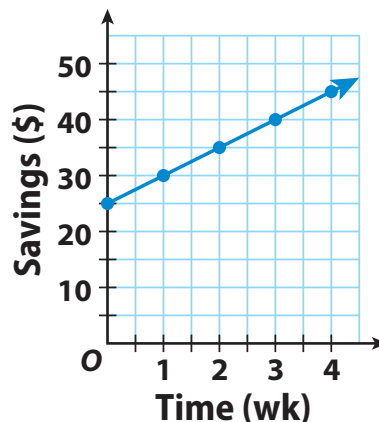
EXAMPLE 1

Jai is saving to buy his mother a birthday gift. Each week, he saves $5. He started with $25. The equation $y = 5x + 25$ gives the total Jai has saved, y, after x weeks. Draw a graph of the equation. Then describe the relationship.

Use the equation to make a table. Then, graph the ordered pairs from the table, and draw a line through the points.

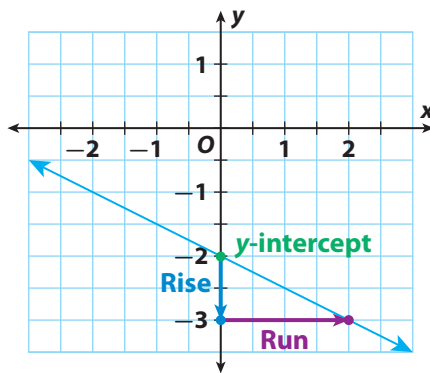| x (weeks)	0	1	2	3	4
y (savings in dollars)	25	30	35	40	45

The relationship is linear but nonproportional.

EXAMPLE 2

Graph $y = -\frac{1}{2}x - 2.$

The slope is $\frac{-1}{2}$, or $-\frac{1}{2}$.

The *y*-intercept is -2.

EXERCISES

Complete each table. Explain whether the relationship between *x* and *y* is proportional or nonproportional and whether it is linear. (Lesson 4.1)

1. $y = 10x - 4$

x	0	2		6
y	−4		36	

2. $y = -\frac{3}{2}x$

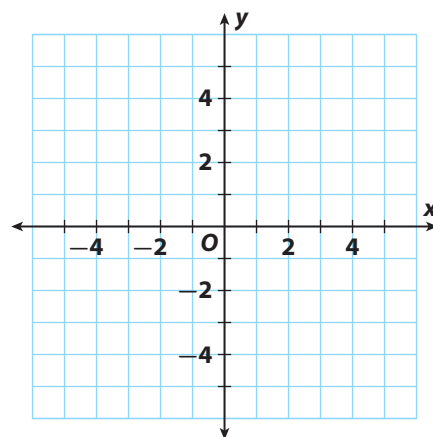| x	0		2	
y		−1.5		−4.5

_____

_____

3. Find the slope and *y*-intercept for the linear relationship shown in the table. Graph the line. Is the relationship proportional or nonproportional? (Lessons 4.2, 4.4)

x	−4	−1	0	1
y	−4	2	4	6

slope _____

y-intercept _____

The relationship is _____.

4. Tom's Taxis charges a fixed rate of $4 per ride plus $0.50 per mile. Carla's Cabs does not charge a fixed rate but charges $1.00 per mile. (Lesson 4.3)

a. Write an equation that represents the cost of Tom's Taxis. _____

b. Write an equation that represents the cost of Carla's cabs. _____

c. Steve calculated that for the distance he needs to travel, Tom's Taxis will charge the same amount as Carla's Cabs. Graph both equations. How far is Steve going to travel and how much will he pay?

_____

_____

Writing Linear Equations
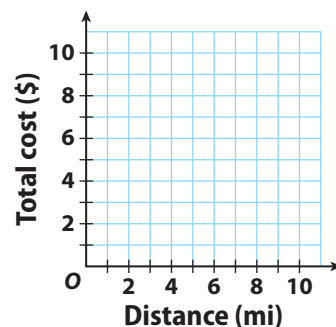
? ESSENTIAL QUESTION

How can you use linear equations to solve real-world problems?

EXAMPLE 1

Jose is renting a backhoe for a construction job. The rental charge for a month is based on the number of days in the month and a set charge per month. In September, which has 30 days, Jose paid $700. In August, which has 31 days, he paid $715. Write an equation in slope-intercept form that represents this situation.

$(x_1, y_1), (x_2, y_2) \rightarrow (30, 700), (31, 715)$ Write the information given as ordered pairs.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{715 - 700}{31 - 30} = 15$ Find the slope.

$y = mx + b$ Slope-intercept form

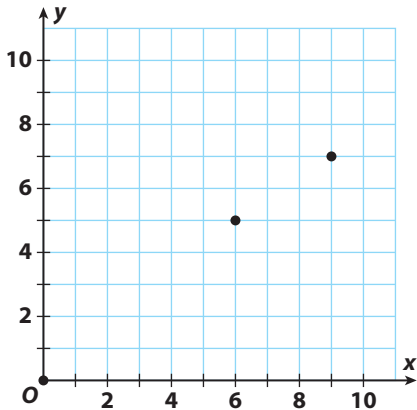$715 = 15(31) + b$ Substitute for *y*, *m*, and *x* to find *b*.

$250 = b$ Solve for *b*.

$y = 15x + 250$ Write the equation.

EXAMPLE 2

Determine if the graph shown represents a linear or nonlinear relationship.

Points	Rate of Change
(0, 0) and (6, 5)	$m = \dfrac{5-0}{6-0} = \dfrac{5}{6}$
(6, 5) and (9, 7)	$m = \dfrac{7-5}{9-6} = \dfrac{2}{3}$
(0, 0) and (9, 7)	$m = \dfrac{7-0}{9-0} = \dfrac{7}{9}$

The rates of change are not constant. The graph represents a nonlinear relationship.

EXERCISES

1. Ms. Thompson is grading math tests. She is giving everyone that took the test a 10-point bonus. Each correct answer is worth 5 points. Write an equation in slope-intercept form that represents the scores on the tests. (Lesson 5.1) _____

The table shows a pay scale based on years of experience. (Lessons 5.1, 5.2)

Experience (years), x	0	2	4	6	8
Hourly pay ($), y	9	14	19	24	29

2. Find the slope for this relationship. _____

3. Find the *y*-intercept. _____

4. Write an equation in slope-intercept form that represents this relationship. _____

5. Graph the equation, and use it to predict the hourly pay of someone with 10 years of experience.

   _____

   _____

   _____

   _____
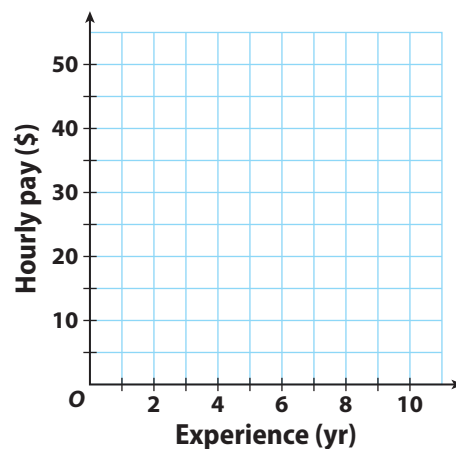
Does each of the following graphs represent a linear relationship? Why or why not? (Lesson 5.3)

6.
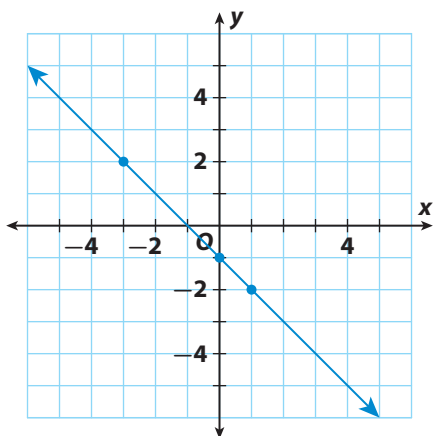
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7.

_____

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? **ESSENTIAL QUESTION**

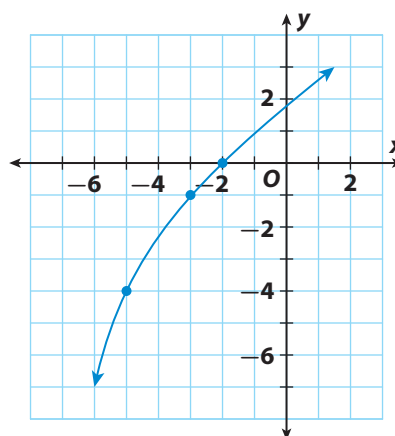How can you use functions to solve real-world problems?

EXAMPLE 1

Determine whether each relationship is a function.

A

Input	Output
3	10
4	4
5	2
4	0
6	5

The relationship is not a function, because an input, 4, is paired with 2 different outputs, 4 and 0.
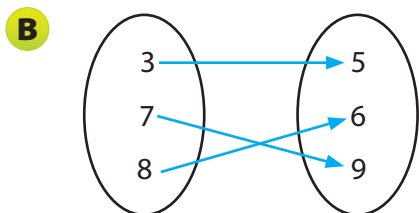
B

Since each input value is paired with only one output value, the relationship is a function.

EXAMPLE 2

Sally and Louis are on a long-distance bike ride. Sally bikes at a steady rate of 18 miles per hour. The distance y that Sally covers in x hours is given by the equation $y = 18x$. Louis's speed can be found by using the numbers in the table. Who will travel farther in 4 hours and by how much?

Louis's Biking Speed			
Time (h), x	3	5	7
Distance (mi), y	60	100	140

Each distance in the table is 20 times each number of hours. Louis's speed is 20 miles per hour, and his distance covered is represented by $y = 20x$.

Sally's ride:

$y = 18x$

$y = 18(4)$

$y = 72$
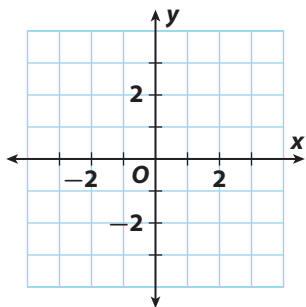
Louis's ride:

$y = 20x$

$y = 20(4)$

$y = 80$

Sally will ride 72 miles in 4 hours. Louis will ride 80 miles in 4 hours. Louis will go 8 miles farther.

EXERCISES

Determine whether each relationship is a function. (Lesson 6.1)

1.

2.

Input	Output
-1	8
0	4
1	8
2	16

_____

_____

Tell whether the function is linear or nonlinear. (Lesson 6.2)

3. $y = 5x + \dfrac{1}{2}$ _____

4. $y = x^2 + 3$ _____

5. Elaine has a choice of two health club memberships. The first membership option is to pay $500 now and then pay $150 per month. The second option is shown in the table. Elaine plans to go to the club for 12 months. Which option is cheaper? Explain. (Lesson 6.3)

Months, x	1	2	3
Total paid ($), y	215	430	645

_____

_____

_____

6. Jenny rode her bike around her neighborhood. Use the graph to describe Jenny's bike ride. (Lesson 6.4)

_____

_____

_____

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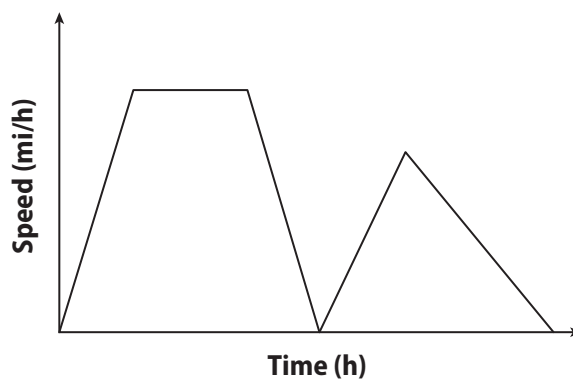
Unit 2 Performance Tasks

1. **CAREERS IN MATH** **Cost Estimator** To make MP3 players, a cost estimator determined it costs a company $1500 per week for overhead and $45 for each MP3 player made.

    a. Define a variable to represent the number of players made. Then write an equation to represent the company's total cost c.

    _____

    b. One week, the company spends $5460 making MP3 players. How many players were made that week? Show your work.

    _____

    _____

    c. If the company sells MP3 players for $120, how much profit would it make if it sold 80 players in one week? Explain how you found your answer.

    _____

    _____

2. A train from Portland, Oregon, to Los Angeles, California, travels at an average speed of 60 miles per hour and covers a distance of 963 miles. Susanna is taking the train from Portland to Los Angeles to see her aunt. She needs to arrive at her aunt's house by 8 p.m. It takes 30 minutes to get from the train station to her aunt's house.

    a. By what time does the train need to leave Portland for Susanna to arrive by 8 p.m.? Explain how you got your answer. As part of your explanation, write a function that you used in your work.

    _____

    _____

    _____

    _____

    b. Susanna does not want to leave Portland later than 10 p.m. or earlier than 6 a.m. Does the train in part **a** meet her requirements? If not, give a new departure time that would allow her to still get to her aunt's house on time, and find the arrival time of that train.

    _____

    _____

    _____

Assessment Readiness

Personal
Math Trainer

Online
Assessment and
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Selected Response

1. Rickie earns $7 an hour babysitting. Which table represents this proportional relationship?

Ⓐ
Hours	4	6	8
Earnings ($)	28	42	56

Ⓑ
Hours	4	6	8
Earnings ($)	28	35	42

Ⓒ
Hours	2	3	4
Earnings ($)	7	14	21

Ⓓ
Hours	2	3	4
Earnings ($)	14	21	42

2. Which of the relationships below is a function?

Ⓐ (6, 3), (5, 2), (6, 8), (0, 7)

Ⓑ (8, 2), (1, 7), (−1, 2), (1, 9)

Ⓒ (4, 3), (3, 0), (−1, 3), (2, 7)

Ⓓ (7, 1), (0, 0), (6, 2), (0, 4)

3. Which set best describes the numbers used on the scale for a standard thermometer?

Ⓐ whole numbers

Ⓑ rational numbers

Ⓒ real numbers

Ⓓ integers

4. Which term refers to slope?

Ⓐ rate of change   Ⓒ y-intercept

Ⓑ equation   Ⓓ coordinate

5. The graph of which equation is shown below?
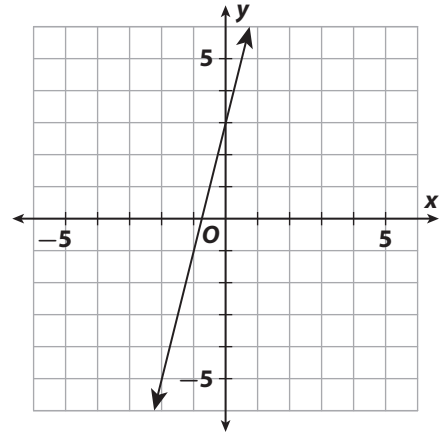
Ⓐ $y = 4x + 3$

Ⓑ $y = -4x - 0.75$

Ⓒ $y = -4x + 3$

Ⓓ $y = 4x - 0.75$

6. Which equation represents a nonproportional relationship?

Ⓐ $y = 5x$

Ⓑ $y = -5x$

Ⓒ $y = 5x + 3$

Ⓓ $y = -\frac{1}{5}x$

7. Which number is 7.0362×10^{-4} written in standard notation?

Ⓐ 0.000070362

Ⓑ 0.00070362

Ⓒ 7.0362

Ⓓ 7036.2

8. Which term does not correctly describe the relationship shown in the table?

x	0	2	4
y	0	70	140

(A) function

(B) linear

(C) proportional

(D) nonproportional

9. As part of a science experiment, Greta measured the amount of water flowing from Container A to Container B. Container B had half a gallon of water in it to start the experiment. Greta found that the water was flowing at a rate of two gallons per hour. Which equation represents the amount of water in Container B?

(A) $y = 2x$

(B) $y = 0.5x$

(C) $y = 2x + 0.5$

(D) $y = 0.5x + 2$

10. Carl and Jeannine both work at appliance stores. Carl earns a weekly salary of $600 plus $40 for each appliance he sells. The equation $p = 50n + 550$ represents the amount of money Jeannine earns in a week, p ($), as a function of the number of appliances she sells, n. Which of the following statements is true?

(A) Carl has a greater salary and a greater rate per appliance sold.

(B) Jeannine has a greater salary and a greater rate per appliance sold.

(C) Carl will earn more than Jeannine if they each sell 10 appliances in a given week.

(D) Both Carl and Jeannine earn the same amount if they each sell 5 appliances in a given week.

Mini-Task

11. The table below represents a linear relationship.

x	2	3	4	5
y	14	17	20	23

a. Find the slope for this relationship.

_____

b. Find the y-intercept. Explain how you found it.

_____

_____

_____

_____

c. Write an equation in slope-intercept form that represents this relationship.

_____

Hot Tip! Estimate your answer before solving the problem. Use your estimate to check the reasonableness of your answer.

12. Jacy has a choice of cell phone plans. Plan A is to pay $260 for the phone and then pay $70 per month for service. Plan B is to get the phone for free and pay $82 per month for service.

a. Write an equation to represent the total cost, c, of Plan A for m months.

_____

b. Write an equation to represent the total cost, c, of Plan B for m months.

_____

c. If Jacy plans to keep the phone for 24 months, which plan is cheaper? Explain.

_____

_____

_____